Final Exam - Review 1 - Problems

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1 Graphing

Problem 1: Graph $y = e^{\frac{1}{x}}$

2 Limits

Problem 2: Evaluate the following limits

(a) $\lim_{x\to\infty} \frac{\tan\left(\frac{1}{x}\right)}{x}$

(b) $\lim_{x\to 0} x^3 \sin\left(\frac{1}{x}\right)$

(c) $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$

(d) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x+9}$

(e) $\lim_{x\to 3} \frac{x^2-6x+9}{x^2-4x+3}$

(f) $\lim_{x\to\infty} \frac{x^2-6x+9}{x^2-4x+3}$

(g) $\lim_{x\to -\infty} x^2 e^{2x}$

(h) $\lim_{x\to 0} (e^x + x)^{\frac{1}{x}}$

3 Derivatives

Problem 3

Using the definition of the derivative, show that the derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$

Problem 4

Show that $f(x) = x\sin(\frac{1}{x})$ is **not** differentiable at 0.

Problem 5: Find the derivatives of the following functions:

(a) $f(x) = \ln(\cos(\sin(\tan(\pi x))))$

(b) The 42^{nd} derivative of $f(x) = e^{2x}$

(c) The equation of the tangent line to $y^3 = x^4 + 8y - 9$ at $(1, 2)$

(d) $f(x) = x^{\cos(x)}$

4 Linear approximations

Problem 6

Use a linear approximation to estimate $\tan(\frac{\pi}{4} + 0.01)$

5 Mean Value Theorem

Problem 7

Show that $x^4 - 5x - 1 = 0$ has at most one zero in $[0, 1]$

Problem 8

If $f(1) = 10$ and $f'(x) \geq -1$ for all x, what is the smallest possible value of $f(5)$?

6 Related rates

Problem 9

A cylindrical gob of goo is undergoing a transformation in which its height is decreasing at a rate of 1 cm/s while its volume is decreasing at the rate of 2π cm^3/s (It retains its cylindrical shape while all of this is happening). If, at a given instant, its volume is 24π cm^3 and its height is 6 cm, determine whether its radius is increasing or decreasing at that instant, and at what rate.

7 Max-Min / Optimization

Problem 10

Find the absolute maximum and minimum of $f(x) = x - 3x^{\frac{2}{3}}$ on $[-1, 27]$

Problem 11

Find the point(s) on the parabola that is (ar) closest to the point $\left(0, \frac{1}{2}\right)$